**Electrodynamics in Insulators**

So we have, assuming constant susceptibilities.

Of course we know that n = n(ω) actually. But we also saw that for small ω much less than any resonant frequency, n(ω) is roughly constant. So we’ll make that assumption here.

**Wave solutions in a polarizable medium**

In isotropic dielectrics, with no free charges, we can write the equations for **E** and **B** as,



Assuming a wave solution of the form,



we get,



So,



The speed of these waves is clearly:



Defining the index of refraction of the material:



we can write this as:



which allows us to assert that the velocity of the wave in the medium is:



Now we lost some conditions by taking derivatives so we’ll plug our solution back into the first equation:



and into the second:



which indicates that the waves are transverse again. Plugging into the third…



we see that the B strength is E/v. So putting it all together we have the same conditions as in the free medium, just changing the velocity from c to v.



The energy density, as we determined from the Insulator Energy-Momentum file, is just:



To take the time average, we could just fill **E**(r,t) into u and integrate over a period. But instead recall the formula from the Free EB Asymptotic Radiation file that if we have two waves A(r,t) = Re[A(r,ω)e-iωt] and B(r,t) = Re[B(r,ω)e-iωt], then the time average of their product is: <A(r,t)B(r,t)> = (1/2)Re[A(r,ω)B\*(r,ω)]. So we have:



Assuming **E**0 is real, this gives us:



Likewise, as in that file, we will find the **intensity** (energy per unit area per unit time transported by such a wave, i.e., energy current density) is:



The time averaged power is easy to get from here, obviously. But let’s use a formula from the Free EB asymptotic radiation file, namely <S> = (1/2)Re(**E**\*(r,ω)×**H**(r,ω)) = (1/2)Re(**E**(r,ω)×**H**\*(r,ω)). Where **E**(r,t) = Re[**E**(r,ω)e-iωt] and **H**(r,t) = Re[**H**(r,ω)e-iωt]. So we have, assuming the amplitudes are real:



where is the direction of **E**×**B**. Using v = 1/√(με). We can write this as:



Can see we can also write <S> = <u>**v**. Also, we have, from the insulator energy-momentum tensor file, and help from the Free EM waves file:



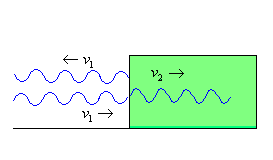
and,



Let’s do a quick example.

**Transmission through semi-infinite polarizable medium (normal incidence)**

Now let’s consider solutions to this equation in 1D. Suppose we set up an **E**-field impinging upon a substance from the left. This is presumed known. There will be a reflected and transmitted wave so…



On the left hand side we have,



And our solution would be, say,



so as to comprise an incident and reflected wave. I keep the time dependence the same – otherwise can’t seem to satisfy the BC. That tells us that the incident, reflected, and transmitted waves all have the same frequency. Plugging this into the equation we find that,



And on the right we’ll have:





Plugging into the equation yields:



and so we can write,



Plugging in the **B**’s would give the same result.

**Boundary Conditions**

Now match up boundary conditions. Since there is no free charge assumed to be present at the interface,  will be 0 across the interface – but we have **E** perpendicular to the interface so this is already satisfied. Additionally, we have **n** × Δ**E** = 0 and so this implies,



which just means that E must be continuous across the boundary. Other boundary conditions are obtained if I introduce **B**. Since there is no free current at the boundary, we have:



Plugging in the relation between **E** and **B** we get,



(we remember that **k**1- points back to the left). So our two equations are:



**Reflection/Transmission Amplitudes**

Now we can define the reflection and transmission amplitudes, just like we do for QM.



Written in these terms, our equations become:



These can be rearranged to:



with solutions,



where in the ≈ step we used the fact that μ ≈ μ0 for most materials. So we have:



If we define β = n2/n1, then we can write:



Can observe that r = t – 1.

**Reflection and Transmission Coefficients**

Observe that as β→ ∞, the reflected wave becomes completely inverted w/r to the transmitted wave. And now, the actual transmittance, and reflectance is proportional to the intensity that makes it through.



Refering to our formula for **S** above, we have,



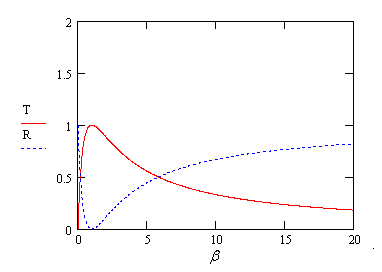
where we assume μ1 ≈ μ2 ≈ μ0. And similarly,



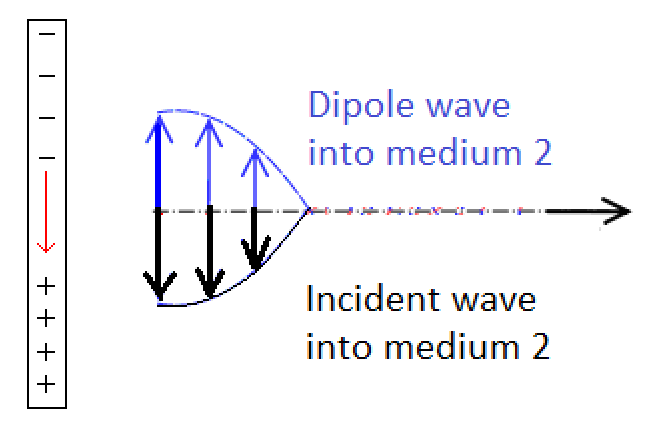
Therefore we have:



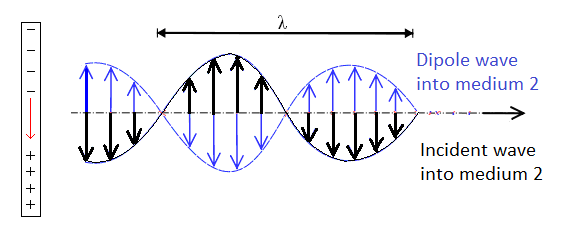
So when β = 1, we have T = 1, R = 0. But when β is large T -> 0, but R -> 1. This is because when β is small, the refractive medium has low polarizability, and so barely interferes with the transmission of the wave. When β is large, the refractive medium has high polarizability, and so the electrons/charges oscillate with large amplitudes (always in phase with the field), and allow very little of the field to transmit, reflecting most of it. Consider plots of T and R as a function of n.



Let’s consider the case of incidence on a higher index of refraction, so n2>n1, i.e., β = n2/n1 > 1. We see that as the index of refraction increases, the fraction of light that is reflected increases as well. We can surmise this is so because as n increases, the polarizability of the dipoles increase. Therefore their oscillations in the electric field will increase. Therefore their acceleration will increase, and so the amplitude of the waves that they emit will increase. Thus we’ll have a relatively large reflected wave going back into medium 1. Could say it (dipole) will also produce a large transmitted wave into medium 2, which will be 180o out of phase with the incident wave traveling into medium 2, so that the net wave traveling into medium 2 will be small. Tried to convey that below. I draw the dipole wave as being same amplitude as incident wave, though it would be smaller in fact so as not to produce complete cancelation.



But can see that the dipole transmitted wave will be 180o out of phase with the incident wave since when the dipole is oriented as shown, positive charges on bottom, negative on top, its dipole wave will be oriented upwards. But at the same time, in order for the dipole to have obtained this orientation, the incident field must have been pointing down, to have already pushed the positive charges down. So they are completely out of phase. Note that in our time-independent model, there is no phase difference between the orientation of the incident field and the orientation of the dipole. Might as well point out that there is no absorption of our incident wave – it’s either transmitted w/o attenuation into medium 2 or reflected back into medium 1. Cause for a λ/4 period, the incident EM field is up, and dipole (+) charge is moving up. During this time the field is stretching the molecular bond, giving it PE. But then over the next λ/4 period, the incident EM field is still up, but dipole (+) charge is moving back down. So now the potential energy is going back into the EM field. So the incident EM field (in all its manifestations: incident, reflected, transmitted) doesn’t lose any energy.



On the otherhand, if we consider what happens upon incidence with a smaller n index of refraction (n < 1), then we see that transmittance and reflection behaves symmetrically about n = 1. In fact it is completely symmetric in the sense that the waves obey time reversal symmetry so that T(n2/n1) = T(n1/n2), and you can verify that T(n) = T(1/n) as expected.